Western Carolina University

Jamaican Program

Course Title	Math 322 – Theory of Arithmetic II
Instructor	Delbert Greear
Dates	Class dates from Oct 8 to Oct 21, 2014 (With additional time for post assignment…to be decided with class discussion).
Contact Info:	dgreear@ung.edu ; or delbert.greear@ung.edu ; or Work Phone: 678-717-3736;
Text	Modern Mathematics by Wheeler and Wheeler, 13th edition Kendall/Hunt
Materials:	Standard classroom materials and a calculator. There will be also some geometry, requiring a drawing compass and protractor/straight-edge. Some graphing calculators for group work may be available, unless they have been pre-empted by other classes.

**** PRE-COURSE ASSIGNMENT ** (Found on page 7)**

This course requires that students complete a pre-course assignment before the instructor arrives.

Course Description

A study of the mathematical development of the Rational number system. *Prerequisite: Math 230(321) or equivalent.*

Course Overview

The purpose of the 321-322 course sequence is to provide a theoretical basis for the structure of the real number system. The intent is to provide the prospective elementary school teacher an appropriate background for teaching mathematics in the primary grades. Basic concepts of the mathematical systems of whole numbers, integers, and rational numbers will be stressed. Also, fundamental topics in problem solving, number theory and mathematical applications will be included. Understanding of number theory is enhanced by study of set theory, logic, geometry, and algebra, which are equally strong components of quantitative reasoning needed by inhabitants of today's world.

Objectives: In this course the student will be asked to;

1. Apply fundamental properties of inductive and deductive reasoning;
2. Illustrate and test mathematical properties related to each of the four basic operations—addition, subtraction, multiplication, and division;
3. Illustrate and test the properties of the operations described in objective 2 as related to whole numbers, integers, and rational numbers;
4. Use various models to illustrate concepts and skills (including algorithms) for each number operation;
5. Use manipulatives or drawings to represent course concepts or processes;
6. Experience real world applications of course topics.

Jamaican Relevancy

Because few resource rooms exist in Jamaican schools to address the needs of students with mild disabilities (learning disabilities, ADHD, slow learners, poor readers, etc.) general education teachers are faced with addressing the needs of a diverse group of students with multiple learning and emotional needs. This course focuses on students with mild-moderate disabilities and how their learning can be accommodated in a general classroom environment through effective teaching practices and differentiating instruction. Students will be expected implement strategies discussed in class and report to the class the benefits and/or difficulties encountered when implementing a particular strategy with their students. The final evaluation for the class will include a question related specifically to whether the information presented during the course was 'relevant to Jamaican schools'.

Diversity Statement

Diversity in our society and schools is an important part of our history and a significant variable in discussions regarding the aims and purposes of education and schooling. The content of this course reflects a commitment to social justice and an emphasis on the necessity of preparing educators who will effectively meet the individual needs of ALL students. As such, issues related to race, gender, religion, sexual orientation and social class will be considered in this class. Students in this class teach in Jamaican schools. Jamaica is a diverse nation (the motto: "Out of many, one people" exemplifies a commitment to and respect for our varied backgrounds, beliefs and values).

Accommodations for Students with Disabilities

Western Carolina University is committed to providing equal educational opportunities for students with documented disabilities. Students who require disability services or reasonable accommodations must identify themselves as having a disability and provide current diagnostic documentation to Disability Services. All information is confidential.

Policy on Academic Honesty

The use in any way of the work of another person or source to fulfill the requirements of this course is academically dishonest and grounds for removal from this class, the program, and possibly the university. Of course with group projects everyone is expected to participate in the answering of the questions. In such cases all will get the same grade unless an individual fails to participate adequately to their ability.

Attendance/Participation

Active participation and regular attendance are necessary for completion of the requirements of this course. In addition, much of the course is active learning, and many core understandings will be achieved by participation in class activities, discussions, and group assignments.

Projected Course Schedule

(Note: Page numbers and items listed are from the Wheeler text and this homework is to be discussed the next class period. There will be additional assignments for the Group Assignment Notebook (henceforth GAN) that is due later in the summer about a week after the course. These assignment notebooks will be turned in by each group. All group members get the same grade on group exercises unless there is reason to believe a group member is contributing less than his or her fair share.)

Please see last page for pre assignment as well! Note that specific homework is subject to amendments and change as we go along.

Course Assignments:

1. Attend all classes.
2. Read and complete all text assignments and participate in class activities/discussions.
3. Successfully complete exams/quizzes.
4. Complete a Homework Assignment Notebook Project to be submitted to the instructor by Jun 28, 2010.

Evaluation:

Pre-assignment Work (Individual Grade) ….……5%
Exams/Quizzes (Take Home and In Class) ……35%
Final ……...20%
Group Assignment Notebook (GAN) (Group Grade) ……35%
Class Participation (Includes Attendance and Discussion Contribution Grade) ……5%
Grading Scale: A 90-100; B 80-89.49; C 70-79.49; D 60-69.49; F < 60

CLASS CALENDAR: (PRELIMINARY--SUBJECT TO CHANGE AS WE GO ALONG)

Day 1 – Oct 8	• Course Introduction;
	• **2.1: Introduction to Logic:** p. 40+: 1-6 (1 through 6),11, 12, 18;
	  ○ **GAN Problem 1: TBA (To be Announced.)**
	• **2.2: Conditionals & Equivalencies:** p.49+1, 2, 3, 6, 13, 14;
	• **3.1 Introduction to Sets:** p. 79+: 1-8,12,13,16;
	• **2.3(Logical Arguments) and 2.4(Quantifiers, Venn Diagrams,Valid Arguments)** *if time*
	• **Review: 1.1/1.2 Patterning;** p. 12+: 13, 14, 17,18 ,28;
	• **1.3 Intro to Prob. Solv.** : (Ex. Set 2) p. 21+ 15, 16, 21,
	  ○ **GAN Problem 2: TBA**

Day 2 – Oct 9:	• **More about Sets & Problem Solving:** Just for fun page 95.
	• **Section 3.2: Cartesian Product**: (Brief Overview) Cartesian Product and Relations, Special relation on AXA: reflexive, symmetric, transitive relations. Equivalence relation has allthree properties. Distributive property of relation across union and intersection (p87): p89+:2, 3, 4, 8 11,12
	• **3.3: Elements in sets & subsets:** cardinal number of a set, equivalent sets, one-to-one correspondence, cardinal, ordinal, and nominal #'s, $n(A)$, $n(A \cup B)$, $n(A \cap B)$ p. 96+: 1, 4, 5, 9, 11, 12;
	  ○ **GAN Problem 3: TBA,**
	• **Section 3.4 "Functions":** Definitions, paradigms, function notation. **Function as a subset of relations.** p104+: 1, 2,3, 4, 5, 6;
	  ○ **GAN Problem 4: TBA:**
	  ○ **GAN Problem 5: p109 "Letter Digit Addition"**
	• **Go over "function notation".**
	• **4.1: Whole # + and – and order:** Addition of whole numbers, closure of W under add, commutative & associative properties, zero serves as "additive identity", subtraction of whole numbers, minuend and subtrahend, models (paradigms) take-away, comparison, missing addend, ordering the whole numbers (p119), solving simple equations as "open sentences" solutions and "solution sets" (p120): HW: p122+: 1-4, 6, 11-13, 22
	• **4.2: Whole # x and ÷**: Models and paradigms for multiplication: repeated addition, geometric area of rectangle, Def. by cardinal # of Cartesian product of two finite sets (p128), Commutative and Associative properties, 1 is "multiplicative identity", zero is multiplicative "destroyer", distributive property of mult. Over addition (and subtraction):: Division as opposite or inverse of mult.,div of whole numbers (p133), dividend, divisor, quotient (remainder may be mentioned), models and paradigms: partition, measurement, missing factor, repeated subtraction, zero as dividend (ok), zero as divisor (not ok), 0/0 is "indeterminate", division as a function f(x)=x/a, $a \neq 0$, standard order of operations (p135), HW: p138+: 1-7, 14,

	15 23, 30 ○ **GAN Problem 6: (Game of "24") Arrange the four whole numbers, 2, 4, 5, 9 in any order, inserting parentheses, and arithmetic operation signs $+, -, *, \div$ in such a way as to create the number 24. For example, given the numbers 3, 6, 8, 10, one could write: $(10-(8-6))*3=24$, and there may be other answers of course. For example in this last case one could also write: $(10-8+6)*3=24$.** ○ **GAN Problem 7: TBA** ○ **Take up pre-assignment** **NOTE: Friday Oct 23 is Graduation for a previous cohort, so Class will not be held that day, instead there will be a long weekend.**
Day 3 – Oct 13:	• Go over the concept of the "open sentence." Example: (1) "He is a gangster."(2) $3x-5=13$ (an open sentence is a statement that is true for a certain subset of values. (1) is true for Al Capone and dudus Coke, (2) is true for x=6. • **4.4: Place Value:** powers of ten, exponents and exponent rules (p 153), other bases (base 7, base 2, base 3, base 8, base 12…these are some bases used in mathematics for various reasons.) See page 159 for "duodecimal" (base 12) discussion. **HW: p 160+ 1-10, 20, 27** • **4.5 + and – Algorithms**: Brief mention, addition tables in other bases, **HW: p172+ 15, 16, 18, 26** • **4.6 x and ÷ Algorithms:** Partial products (p175), Note algebraic expansion: $(a+b)(c+d)=ac+ad+bc+bd$ Memorized b "FOIL" mnemonic, (done with 13*14 in example 3 p175), Lattice multiplication (p177), Russian Peasant Multiplication (p177), converting numbers to base two by RPM (multiply the number by 1):: standard Division Algorithm (with remainder p178) $a=bq+r$, "scaffolding" (p179)…actually this is based on repeated subtraction, multiplication tables for other bases (p181), **HW: p184+ 4, 33, 34, 35, 36.** ○ **GAN Problem 8: Just For Fun p183** ○ ****GAN Problem 9: History tells us that Wild Bill Hickock was holding a five card poker hand with a pair of aces and a pair of eights when he was shot to death in a bar in Deadwood South Dakota. History is silent as to the fifth card, or "kicker" that he held. Can Wild Bill "get 24" with his ace and eights, and if so how?** ○ ****GAN problem 10: GAN Problem 7: TBA** ○ **Take up pre-assignment** ○ ****GAN Problem 11: Is the number $242_{(seven)}$ an even number? Will a base seven number ending in an even digit always be even? Will a base seven number all of whose digits are even be an even number? Explain you answer** ○ **GAN Problem 12: Multiply 14 by 850 by means of the Russian Peasant Multiplication algorithm.** ○ **GAN Problem 13: Find the binary representation of 1400 by using Russian Peasant Multiplication of 1400 *1 , doubling the 1's column and halving the 1400's column. Check with your calculator.** • **5.1: Intro to Integers:** additive inverses, positives, negatives, absolute value (page198), subtraction, **HW: p 202+ 5, 6, 10, 11, 18**
Day 4—Oct 14:	• **Review 5.1 Integers + and –: Go over definition of absolute value p198** • **Review 4.6: Division Algorithm p178: (need this for Euclean algorithm)** • **5.2 Integer Multiplication and Division:** Mainly positive * positive=negative * negative =positive, oddnumbrer of negatives multiplied together yields a negative, even number of negatives multiplied together = positive. Also zero is an even

	number. **HW 5.2 p209+: 1, 2, 5, 8, 13, 14, 16, 17** • **5.3 Number Theory: (Integer Multiplication and division):** "Divides" definition p212, vertical line for divides, skew line for fraction, $a \mid b$ vs b/a , properties of divisibility p213, do example 6 (proof), Some divisibility criteria p214 (base 10) divisibility by a product p216, Primes and sieve of Eratosthenes p216, definition of prime & square root rule for checking for primality p217, question why is one not prime?, composites definition p218, Fundamental Theorem of Arithmatic p219, Open questions in Number Theory: perfect, deficient, abundant & amicable numbers, Godbach's conjecture (not mentioned in text), HW: p222+ 1-10, 12-14, 21, 22, 24, 26 • **5.4 gcd and lcm or GCF & LCM:** Greatest Common Factor p225 largest whole number that divides a set (pair) of numbers, product of largest powers of primes in common, concept of "relatively prime" p 227, Euclidean Algorithm $GCF(a,b) = GCF(b,r)$ where *r* is the remainder from division algorithm , least common multiple p230, "smallest integer divisible by two numbers" in terms of abs value, also product of highest powers of primes in either of the two numbers, formula p232 $LCM(a,b) = \frac{ab}{GCF(a,b)}$, Least Common Denominator (needed for dding fractions) is least common multiple of the denominators of fractions (p233):: HW 5.4: p235+ 1-6, 8-11, 14, 18, 21 ○ ****GAN Problem 14: On the distant planet Zarcon, the number system is based on 13, with digits 1, 2,…8, 9, T, E, W . The number WWWW is considered sacred. What is its representation in base 10? In base 12? Is this number prime?** ○ ****Gan Problem 15: GAN Problem 7: TBA** ○ **Take up pre-assignment**
Day 5 – Oct 15:	• **Go over Pre-Assignment, & Quiz 1, esp.** ○ Problems: Train in Tunnel, chickens and eggs (set 2: 20 & 22) ○ Problems: 3, & one of set 2, ○ Take up Quiz 2 • **Go over work to date, especially 5.4.:** Fundamental Theorem of Arithmetic • **6.1 Rationals/Fractions:** Definitions, Models and Paradigms p251 ○ 1. Part of whole. 2. Part of set. 3. Point on number line. 4. Division of whole #'s. ○ Definition of rational numbers (p251) How does this differ from a "fraction", (difference is subtle). Also every rational # represents a unique point on number line. Equivalent fractions (equivalence relation on the rational numbers). Fundamental Law of Fractions (FLF) (p254). Simplest form (p255) $\frac{a}{b} = \frac{c}{d} \Leftrightarrow ad = bc$, $b \& d \neq 0$, converting fractions to different forms using FLF, Equal and un-equal fractions (p256), Ordering the rationals (p258) $\frac{a}{b} < \frac{c}{d} \Leftrightarrow ad < bc$, $b \& d > 0$ NB: the denominators must be positive. Subtle difference between fractions and rational numbers: last sentence of the chapter p259. TI 85 Arrow Fraction Key. **HW: P259+: 1-6, 9, 10, 13, 15, 25**. • **6.2 Fraction + and –,** Briefly, to add and subtract fractions, one must find a common denominator, then add and subtract the numerators. The most convenient of the common denominators is usually the LCD, which is the least common Multiple of the Denominators. Usually, "improper fractions are easier to deal with in algebraic context, not always true for everyday arithmetic, where a "mixed number" might be better…or

	possibly in locating number on the number line. Note estimation strategies on p271**. HW: p273+ 5, 6, 7, 20** . • **6.3 Fraction x and ÷:** Geometric Paradigm, Rule for multiplication (p277), in practice, cancellation frequently makes things easier. "Multiplicative Inverse" (p279) exists for each rational number with non-zero numerator: $\frac{a}{b}*\frac{b}{a}=1$, $a \& b \neq 0$ (Remember an additive inverse for a is $-a$ such that $a+(-a)=0$ so each rational number also has an additive inverse. ? Does zero have an additive inverse? To divide fractions, invert the divisor and multiply (p281). Some algebra: Multiply mixed numbers (p279): $(a+b)(c+d)=ac+ad+bc+bd$ Ex: $\left(3+\frac{1}{2}\right)\left(5+\frac{1}{3}\right)=?$ **HW: p285: 1-5, 7,8, 11, 30, 34** . ○ **GAN Problem 16: Igor the Russian Peasant likes to do things by threes instead of by twos. He multiplies $31*42$ like this:** 31 42 10 126 ~~3~~ ~~378~~ 1 1134 1302 **Describe his method. Try it for other problems, starting with $30*42$. Does it appear to be sound? Can this type of Russian Peasant Multiplication be used to convert base ten numbers to base three?** ○ **GAN Problem 17: GAN Problem 7: TBA** ○ **GAN Problem 18: The numbers 100 and 10,000 are perfect squares. Are the numbers $100_{(seven)}$ and $10,000_{(seven)}$ also perfect squares? Explain.** ○ **GAN Problem 19:** Ivan crosses out the rows with odd numbers in the first column by accident. Correct his mistake, if possible. *XXXXXXXXXXXX* *XXXXXXXXXXXX* 8 312 4 624 2 1248 *XXXXXXXXXXXX* 2184 ○ **Gan Problem 20:** **Discuss the advantages and disadvantages of : a. Russian Peasant Multiplication b. Lattice Multiplication c. The "Modern Algorithm" for multiplication.
Day 6 – Oct 16	**Catch up**
Day 7– Oct 17:	6.4 Ratio/Proportion, Dimensional Analysis , - Catch-Up, - Group Assignment Notebook - Project Plans
Day 8 – Oct 20:	- Final Review - Grade Report Schedule Discussion
Day 9 – Oct 21:	**Final Exam**

PRE COURSE ASSIGNMENT:

This course requires that students complete a pre-course assignment before the instructor arrives. This assignment asks you to address the topic of "Critical Thinking and Inductive Reasoning" and the associated subject of "Problem Solving" (which plays prominent role in the modern elementary mathematics curriculum.) Your work related to this assignment should be written and submitted on the first day of class.

Assignment: Read Sections 1.1, 1.2, 1.3, & 1.4. Critical Thinking and Problem Solving of the text listed below; then answer the following: (Note: Questions marked with asterisks (**) require complete sentence answers for full credit…this is a convention I use on all tests and turned in work. This helps to be sure that the student understands the problem, and that the professor understands the student. Thanks for your cooperation.)

A. **Page 12+(Exercise Set 1 (Sections 1.1 & 1.2)): 2,3,6,7,8,9,10,11,12,14,15,16,17,19, 20,22,**25**
B. **Page 21+ (Ex. Set 2) (Section 1.3): 1,3,3a:make up a similar problem to 3 using Jamaican Currency, solvable by a 5th grade level or below,6,7,**9,15,19,**21**
C. **Page 28 (Ex. Set 3) (Section 1.4): 7,8,**10,17,19,20,20a: Make up a similar problem to 20 or 21 that you think a 5th grader or below could solve.),**28 (In 28, it is assumed that all the labels are wrong…ie. none are correct),**

If you are unable to get a solution to a problem tell (write down) how you are attempting to solve it and describe any progress toward reaching a solution; or, describe what specifics about the problem are blocking you from reaching a solution.

D. **How confident are you in your understanding of mathematics?**
    1. **What areas were easy for you?**
    2. ** What areas were difficult, that later you mastered?**
    3. **What are some areas that you still find difficult to understand?**
    4. **What are some areas you would like for us to concentrate on in this course**