MATH REVIEW & NEW CONCEPTS NAME: Answer Key

Algebra (Review)

1. Given the equation $x = vt + \frac{1}{2}at^2$, solve for the indicated variable. Show your work. (1 point each)

(a) Solve for v.

$$x = vt + \frac{1}{2}at^2$$

$$vt = x - \frac{1}{2}at^2$$

$$\boxed{v = \frac{x - \frac{1}{2}at^2}{t}} \quad \text{OR} \quad \boxed{v = \frac{x}{t} - \frac{1}{2}at}$$

(b) Solve for a.

$$x = vt + \frac{1}{2}at^2$$

$$\frac{1}{2}at^2 = x - vt$$

$$a = \frac{2(x - vt)}{t^2}$$

$$\boxed{a = \frac{2x - 2vt}{t^2}} \quad \text{OR} \quad \boxed{a = \frac{2x}{t^2} - \frac{2v}{t}}$$

(c) Solve for t.

$$x = vt + \frac{1}{2}at^2$$

$$\frac{1}{2}at^2 + vt - x = 0$$

$$a = \frac{1}{2}a; \; b = v; \; c = -x$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-v \pm \sqrt{v^2 - 4(\frac{1}{2}a)(-x)}}{2(\frac{1}{2}a)} = \boxed{\frac{-v \pm \sqrt{v^2 + 2ax}}{a}}$$

2. Given the equation $F = {mv^2}/{r}$, solve for r if $F = 455, m = 94$, and $v = 22$. (1 point)

$F = mv^2/r$

$455 = \frac{94(22)^2}{r}$

$455r = 94(22)^2$

$455r = 45{,}496 \longrightarrow r = \frac{45{,}496}{455} = \boxed{100}$

3. Given the equation $K = \frac{1}{2}mv^2$, solve for v if $K = 42$ and $m = 9.1$. (1 point)

$K = \frac{1}{2}mv^2$

$42 = \frac{1}{2}(9.1)v^2$

$42 = 4.55v^2$

$v^2 = \frac{42}{4.55} = 9.23 \longrightarrow v = \sqrt{9.23} = \boxed{3.0}$

4. Solve the following system of equations for v_1 and v_2, given $m = 5$ and $v_f = 2.1$ (2 points):

$$\frac{1}{2}m{v_1}^2 + \frac{1}{2}m{v_2}^2 = m{v_f}^2$$

$$mv_1 + mv_2 = 2mv_f$$

$\frac{1}{2}(5)v_1^2 + \frac{1}{2}(5)v_2^2 = (5)(2.1)^2$

$5v_1 + 5v_2 = 2(5)(2.1)$

$2.5v_1^2 + 2.5v_2^2 = 22.05$

$5v_1 + 5v_2 = 21$

$5v_1 = 21 - 5v_2$

$v_1 = \frac{21 - 5v_2}{5}$

$v_1 = 4.2 - v_2$

$2.5(4.2 - v_2)^2 + 2.5v_2^2 = 22.05$

$2.5(17.64 - 8.4v_2 + v_2^2) + 2.5v_2^2 = 22.05$

$44.1 - 21v_2 + 2.5v_2^2 + 2.5v_2^2 = 22.05$

$5v_2^2 - 21v_2 + 22.05 = 0$

$v_2 = \frac{21 \pm \sqrt{(-21)^2 - 4(5)(22.05)}}{2(5)}$

$v_2 = \frac{21 \pm \sqrt{441 - 441}}{10} = \frac{21}{10} = \boxed{2.1}$

$v_1 = 4.2 - 2.1 = \boxed{2.1}$

SI Units & Prefixes (New) (1 point each)

4. Express 3.58 milligrams (mg) numerically in grams (g).

0.00358 g

5. Express 7840 centimeters (cm) numerically in kilometers (~~m~~ km).

0.0784 km or 78.4 m

6. Express 0.26 kilojoules (kJ) numerically in joules (J).

260 J

Exponents, Scientific Notation, and Significant Figures (New) (½ point each)

7. Simplify each of the following:

(a) $x^3 \cdot x^5$

x^{3+5}

$\boxed{x^8}$

(b) $10^7 \cdot 10^{-3}$

10^{7-3}

$\boxed{10^4}$

(c) $(2.5 \times 10^{-6})(4 \times 10^6)$

$10 \times 10^{-6+6}$

$10 \times 10^{0} \rightarrow 1$

$\boxed{10}$

(d) $\frac{(6\times10^6)(4\times10^{-5})^4}{(8\times10^2)^2(2\times10^{-4})^3}$

$$\frac{(6\times10^6)(4^4\times10^{-5(4)})}{(8^2\times10^{2(2)})(2^3\times10^{-4(3)})}$$

$$\frac{(6\times10^6)(256\times10^{-20})}{(64\times10^4)(8\times10^{-12})}$$

$$\rightarrow \frac{1536\times10^{6-20}}{512\times10^{4-12}}$$

$$\frac{1536\times10^{-14}}{512\times10^{-8}}$$

$$3\times10^{-14+8}$$

$$\boxed{3\times10^{-6}}$$

8. Express each of the following using scientific notation:

(a) 32,600

3.26×10^4

(b) 0.831

8.31×10^{-1}

(c) 0.0002

2×10^{-4}

(d) 1006

1.006×10^3

(e) 0.00000000019

1.9×10^{-10}

(f) 299,000,000

2.99×10^8

9. How many significant figures are in each of the following?

(a) 130.0

4

(b) 2500

2

(c) 0.04569

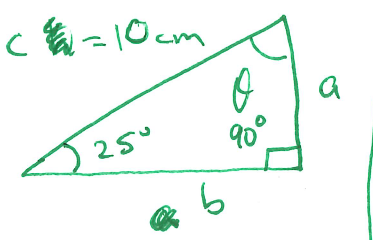
4

(d) 6.50 x 10^{-7}

3

Trigonometry (Review)

10. You are given a right triangle with one acute angle of 25° and a hypotenuse of length 10 cm. Find the other acute angle and the lengths of the other two sides. (3 points)



$\sin \theta = \frac{opp}{hyp}$

$\sin(25°) = \frac{a}{c}$

$a = c \cdot \sin(25°)$

$a = 10 \cdot \sin(25°) = \boxed{4.23 \text{ cm}}$

$\cos \theta = \frac{adj}{hyp}$

$\cos(25°) = \frac{b}{c}$

$b = c \cdot \cos(25°)$

$b = 10 \cdot \cos(25°) = \boxed{9.06 \text{ cm}}$

$\theta + 25 + 90 = 180$

$\theta = 180 - 90 - 25$

$\boxed{\theta = 65°}$

Vectors (New)

11. An object is located at a position of (-4, 7) on a Cartesian coordinate system. Find the magnitude and direction (given as an angle between 0° and 360°) of its position vector. (2 points)

$r = \sqrt{x^2 + y^2}$

$r = \sqrt{(-4)^2 + 7^2}$

$r = \sqrt{16 + 49}$

$r = \sqrt{65} = \boxed{8.06}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$\theta = \tan^{-1}\left(\frac{7}{-4}\right)$

$\theta = \tan^{-1}(-1.75)$

$\theta = -60.26°$

angle should be in 2nd quadrant, so

$\theta = -60.26° + 180°$

$= 119.7°$

$\approx \boxed{120°}$

12. An object moves from its original position of (5, 2) to its new position of (8, -3). (1 point each)

(a) Find the *change* in position and write this as a vector.

$(8, -3)$
$-\ (5, 2)$

$\boxed{(3, -5)}$ OR $\boxed{\langle 3, -5 \rangle}$

(b) What is the magnitude of the vector found in part (a)?

$r = \sqrt{x^2 + y^2}$

$r = \sqrt{3^2 + (-5)^2}$

$r = \sqrt{9 + 25}$

$r = \sqrt{34} = \boxed{5.83}$

(c) What is the direction (given as an angle between 0° and 360°) of the vector found in part (a)?

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$\theta = \tan^{-1}\left(\frac{-5}{3}\right)$

$\theta = -59°$ ← not the correct answer

our vector is in quadrant IV, so we need to add 360° to get a positive angle in the correct quadrant:

$\theta = -59° + 360° = \boxed{301°}$