

# MATH 135

## Statistics, Functions, and Rates of Change

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# 1 Lecture Notes

## 1.1 Selected terms from Chapters 1 and 2

a) numerical facts

b) a discipline of study - methods used to collect, analyze, present, and interpret data, and to make decisions and inferences based upon that data.

Population -

Element or Member of the population -

Sample -

biased sample -

representative sample -

random sample -

stratified sample -

Variable -

categorical (qualitative) -

numeric (quantitative) -

discrete -

continuous -

Descriptive vs. Inferential statistics -

Some methods of displaying data:

frequency distribution  
pie chart  
bar graph/histogram  
stem-and-leaf display  
boxplot (box and whisker plot)

Some shapes of distributions:

unimodal  
bimodal  
symmetric  
positively or negatively skewed

Outlier -

## 1.2 Functions

Function:

Domain:

Range:

May be expressed many ways, such as pictorially, as tables, as ordered pairs, or graphically

...

Example of a function expressed several ways:

Domain:

Range

Many functions can be expressed by an equation

Example of an equation, a partial table of values, and its graph

Examples of relationships which are NOT functions:

The “vertical line test”:

Sample problems:

1. Consider the set of ordered pairs  $\{(1, 1) (2, 2) (3, 3), (4, 4) (5, 5)\}$

a. Make a graph of the points on the x-y plane

b. Is this a function?

c. What is the domain? the range?

2. By plotting several points and connecting them with a reasonable curve, sketch a graph

of the function defined by the equation  $y = x^2$ .

a. Verify (using the vertical line test) that this is a function.

b. What is the domain? the range?

3. Sally and Samantha live on the same floor of their residence hall. Their rooms are 50 feet apart. Walking at a constant rate, Sally walks from her room to Samantha's room in 15 seconds.

a) Make a graph representing Sally's distance from her room as a function of time

b) Make a graph representing Sally's distance from Samantha's room as a function of time

### 1.3 Function notation

Shorthand notation for “y is a function of x”:  $y = f(x)$

“ $f(x)$ ” is read “the  $f$  of  $x$ ”

- $x$  is the input variable (usually called the “independent” variable)
- $y$  is the output variable (usually called the “dependent” or the “response” variable)

Examples

$$f(x) = 2x + 4$$

$$f(3) = \qquad f(10) = \qquad f(0) = \qquad f(s) =$$

$$g(x) = \frac{1}{2}x^2 - 1$$

$$g(2) = \qquad g(0) = \qquad g(-4) = \qquad g(z) =$$

Piecewise defined function

$$h(x) = \begin{cases} 2x & \text{if } x \leq 0 \\ x^2 & \text{if } x > 0 \end{cases}$$

$$h(-2) = \qquad h(3) = \qquad h(0) =$$

Make a sketch of the function  $f(x) = 2x$

Make a sketch of the function  $f(x) = \begin{cases} 1 & \text{if } x < 0 \\ 2x & \text{if } x \geq 0 \end{cases}$

For the function whose graph is shown at the right,

the domain is

the range is

$f(0) =$                       and  $f(4) =$

$f(\quad) = 4$  (fill in the blank)

## 1.4 Linear functions, part 1

Determine the next 2 values in each of the following tables. Then explain the pattern that you see in the numbers.

x	y	x	y	x	y
0	3	0	3	0	0
1	6	1	7	1	1
2	9	2	11	2	4
3	12	3	15	3	9
4	15	4	19	4	16
5		5		5	
6		6		6	

The first two tables (but not the third) represent linear data. A linear function is a function whose graph is a line.

Recognizing linear data from a table (like above):

Note the pattern in the second table:

$$f(0) = 3$$

$$f(1) = 3 + 4$$

$$f(2) = 3 + 4 + 4 = 3 + 2(4)$$

$$f(3) = 3 + 4 + 4 + 4 = 3 + 3(4)$$

$$f(4) = 3 + 4 + 4 + 4 + 4 = 3 + 4(4), \text{ etc.}$$

In general, a linear function can be written in the form  $f(x) = b + mx$ , or  $y = b + mx$ . (This is known as the slope-intercept form of a line.)

the  $y$ -intercept  $b$  is

the slope  $m$  is

Write linear equations which model the data in the first 2 tables above, and sketch their graphs.

Give a graphical interpretation of the slope and  $y$ -intercept in each case.

Given any two points  $(x_1, y_1)$  and  $(x_2, y_2)$  on the graph of a linear function,

the slope  $m = \frac{y_2 - y_1}{x_2 - x_1}$  (This is frequently called *rise* / *run*.)

Verify that the slope is constant, regardless of the 2 points chosen, in the examples above.

A line goes through the points  $(0, 2)$  and  $(3, 11)$ .  
Sketch the line, find its slope, and write an equation of the line in  $y = b + mx$  form.

In the above problem, you could write the equation because the y-intercept was one of the given points. What if two points are given, neither of which are the y-intercept? Another useful formula is the point-slope formula, which will be derived below.

Let  $(x_1, y_1)$  be a known fixed point on a line.

For ANY other point  $(x, y)$ , the slope  $m =$

Rearranging this equation yields the point-slope formula:  $y - y_1 = m(x - x_1)$ .

Examples:

Find equations of the lines through the following pairs of points

a) through  $(2, 3)$  and  $(4, 7)$

b) through  $(-1, -2)$  and  $(3, 2)$

Andy can produce 10 widgets for \$550, and he can produce 20 widgets for \$600. Write a linear equation relating the number of widgets ( $x$ ) and the production cost ( $y$ ). What is the cost of producing 8 widgets?

## 1.5 Linear functions, part 2

Review: A line has constant slope. The slope  $m =$

Slope of a horizontal line

Example: through  $(2, 3)$  and  $(5, 3)$

Slope of a vertical line

Example: through  $(2, 3)$  and  $(2, 5)$

Sketch:

a) a line with  $y$ -intercept 2 and slope = 3

b) a line with  $y$ -intercept 1 and slope = -1

c) a line with slope  $2/3$

Write a partial table of  $x$ - and  $y$ - values for the first two lines above, and find an equation for each.

(Review: the slope-intercept form of a line is \_\_\_\_\_ )

x	y	x	y
0		0	
1		1	
2		2	
3		3	

Thus...

a positive slope indicates

a negative slope indicates

a zero slope indicates

Review: the point-slope form of a line is

Fill in the missing value in such a way that the following data is linear, and find an equation of the line through the data:

x	y
5	3.3
7	4.4
9	5.5
11	

Word problems:

1. At a certain pizza restaurant, a plain pizza costs \$8, with a charge of \$.75 for each topping. Letting  $x$  represent the number of toppings and  $C$  the cost of a pizza,

a) write an equation for the cost  $C$  as a linear function of the number of toppings  $x$

b) interpret the slope and  $y$ -intercept in the context of this problem

slope

$y$ -intercept

c) what is the cost of a pizza with 5 toppings?

2. It costs \$90 to produce 15 widgets and \$110 to produce 20 widgets.

If the total production cost  $C$  is a linear function of the number of widgets  $x$ :

a) determine  $C$  as a function of  $x$

b) interpret the slope and  $y$ -intercept in the context of this problem

c) determine the cost of producing 50 widgets.

(TIME ALLOWING ...)

3. If we know the temperature of an object in degrees Celsius, we can also determine its temperature in degrees Fahrenheit.

The relationship between the Celsius and Fahrenheit measurements is linear.

The temperature at which water freezes is \_\_\_\_\_ °C and \_\_\_\_\_ °F.

The boiling point of water is \_\_\_\_\_ °C and \_\_\_\_\_ °F.

Letting  $x$  represent degrees Celsius and  $y$  represent degrees Fahrenheit:

determine  $y$  (degrees Fahrenheit) as a function of  $x$  (degrees Celsius), and

determine the Fahrenheit reading of an object which is  $-40^{\circ}$  C.

Linear functions - summary

recognizing linear data in a table -  
(constant slope)

slope  $m =$

significance of positive slope -

significance of negative slope -

significance of zero slope -

slope of a vertical line -

slope-intercept form:

point-slope formula:

## 1.6 Distributive property and factoring (review)

A quick review of the distributive property and factoring

We have already made use of the distributive property:  $a(b + c) = ab + ac$ .  
The outside term  $a$  is multiplied by each term inside the parentheses.

Example:  $10(x + 5) =$

We generalize this principle to a product of binomials:  $(a + b)(c + d) = ac + ad + bc + bd$ .  
In this product, each number in the first set of parentheses is multiplied by each term inside the second set of parentheses.

Examples:

$$(10 + x)(2 + x) =$$

$$(x - 2)(x - 6) =$$

$$(x - 2)(2x + 5) =$$

Finally, note the pattern below:

$$(x + 2)(x - 2) =$$

$$(x + 3)(x - 3) =$$

$$(x + 4)(x - 4) =$$

$$(x + a)(x - a) =$$

\* note well! \*\*

FACTORING - applying the distributive property in reverse.

Examples (binomials with common terms):

$$10x + 5 =$$

$$8x^2 + 3x =$$

$$8x^2 - 4x =$$

Differences of perfect squares - recall (from above) that  $x^2 - a^2 = (x + a)(x - a)$

$$x^2 - 25 =$$

$$z^2 - 9 =$$

$$4z^2 - 9 =$$

Note:  $x^2 + 25$  cannot be factored. (Try it!)

Factoring trinomials

$$x^2 + 5x + 6 =$$

$$x^2 - 5x + 6 =$$

$$x^2 + 5x - 6 =$$

$$6x^2 - 10x + 4 =$$

## 1.7 Percentages (a quick review)

A store manager buys an item for \$100 and marks it up 15%. What is the markup price?

original price:

+ markup amount:

= markup price:

Note that the computation  $100 + (.15)(100)$  can be factored as \_\_\_\_\_  
,  
or  $100(1.15)$  for short.

This illustrates a “quick” way to compute a percentage markup:

multiplying by 1.15 is equivalent to computing a 15% markup

multiplying by 1.25 is equivalent to computing a \_\_\_\_\_% markup

multiplying by \_\_\_\_\_ is equivalent to computing an 8%  
markup.

Another example:

The rabbit population in a certain area is 200. In two years, the population grows by 30%.  
What is the final population (after 2 years)?

The quick way:

Double check (original method):

Now let's consider a markdown. An item has an original selling price of \$100. However, the store is having a "15% off" sale. What is the item's sale price?

original price:

- 15% discount

= sale price.

Note that the computation  $100 - (.15)(100)$  can be factored as \_\_\_\_\_, or  $(100)(.85)$  for short. We will say that .85 is the "multiplying factor" to effect a 15% decrease.

What is the multiplying factor for:

a 10% decrease?

a 25% decrease?

a 5% decrease?

a 1% decrease?

A rabbit population originally consists of 200 rabbits. However, a disease spreads through the colony, wiping out 40% of the population (i.e., the population declines by 40%). What is the final population?

The quick way:

Double check:

## 1.8 Exponents and exponential growth

Suppose that \$1000 is deposited into an account which earns 5% interest each year. Make a partial table of the amount in the account as a function of time (in years):

Number of years (t)	Amount in Account
0	
1	
2	
3	

An account initially has \$1000 in it. However, 5% of the amount in the account is deducted each month. Make a partial table of the amount as a function of time (in months).

Number of months (t)	Amount in Account
0	
1	
2	
3	

Both of these tables represent EXPONENTIAL data (as opposed to LINEAR data). The first table is an example of exponential growth, and the second of exponential decay. The way to recognize exponential data in a table (and when there is growth vs. decay):

In each of the above problems, let time  $t$  be the independent (input) variable, and let  $A$ , the amount in the account, be the dependent (output) variable. Note the following computations:

$$A(0) = 1000 \qquad A(0) = 100$$

$$A(1) = (1.05)(1000) \qquad A(1) = (.95)(1000)$$

$$A(2) = (1.05)^2(1000) \qquad A(2) = (.95)^2(1000)$$

$$A(3) = (1.05)^3(1000) \qquad A(3) = (.95)^3(1000)$$

Letting  $A_0$  be the initial amount and  $b$  be the “multiplying factor” (or “base”), the formula for an exponential function is \_\_\_\_\_

Identify which of the following tables represent exponential data, and write a formula for each of the exponential functions:

t	A	t	A	t	A	t	A
0	10	0	10	0	10	0	10
1	14	1	14	1	8	1	8
2	19.6	2	18	2	6.4	2	5
3	27.44	3	22	3	5.12	3	1

Exponent properties:

$$(2^4)(2^3) =$$

$$(b^x)(b^y) =$$

$$\frac{5^5}{5^3} = \frac{b^x}{b^y} =$$

And we will make the following definitions, which are consistent with the above rules:

$$5^0 =$$

$$b^0 =$$

$$5^{-2} =$$

$$b^{-x} =$$

Graph the exponential functions  $y = 2^x$  and  $y = 10(.5)^x$ .

(Do you recognize these as exponential functions? Determine the initial amount and the base for each function.)

The graphs above illustrate the general shape of exponential functions.

Problem 1: \$2500 is deposited into an account that earns 7.5% annual interest. How much money is in the account after 8 years?

Problem 2: An account earns 6% annual interest. How much money must be deposited now in order to grow to \$656.50 in 4 years?

Compounded interest - earning interest on your interest (as in the above problems).  
Let's explore compounding on a more frequent basis. Suppose you put \$1000 into an account that earns 8% annual interest.

With annual compounding, you earn how much interest?

With semiannual compounding, split that into two halves (4% each 6 months), and give yourself 1/2 year interest on the first \$40.

General formula for interest compounded  $n$  times per year:

$A_0$  initial amount (principal)  
 $r$  interest rate stated on an annual basis  
 $n$  how many compoundings per year  
 $t$  time (number of years money is in the bank)  
 $A(t)$  final amount after  $t$  years

$A(t) =$

Example: \$1000 is deposited into an account earning 6% annual interest compounded quarterly. What does the account grow to after 5 years?

How much money must be deposited now in order to grow to \$2000 after 4 years?

Consider the following table for \$100 deposited into an account earning 6% interest:

compounding frequency $n$	amount after 1 year
1	\$ 106.00
2	\$ 106.09
4	\$ 106.14
12	\$ 106.17
100	\$ 106.18
1000	\$ 106.18
10000	\$ 106.18

Formula for continuous compounding:

\$1000 is deposited into an account earning 6% annual interest compounded continuously.  
How much is in the account after 5 years?

## 1.9 Simplifying rational expressions

$$\frac{x^2y}{x^2z} =$$

$$\frac{4x^2y^3z^4}{8x^4z^4} =$$

$$\frac{4x + y^3}{4x^3y} =$$

$$\frac{6x + x^2z}{xy} =$$

$$\frac{x^2 + 6x + 8}{3x + 6} =$$

$$\frac{x^2 + 6x + 8}{x + 2 + z} =$$

## 1.10 Informal introduction to limits

Consider the function  $f(x) = \frac{1}{x}$ . We will construct a partial table of values and sketch a graph.

$x$	$f(x)$
1/10	
1/2	
1	
2	
4	
10	
100	

As  $x$  gets larger and larger,  $f(x)$  gets closer and closer to \_\_\_\_\_. Observe this feature on the graph.

We say  $\lim_{x \rightarrow \infty} f(x) = 0$ .

Sketch the function  $g(x) = \frac{1}{2^x}$ .

(Note that this is the same as  $2^{-x}$ ; this observation will aid in sketching the function for negative values of  $x$ .)

$$\lim_{x \rightarrow \infty} g(x) =$$

$$\lim_{x \rightarrow -\infty} g(x) =$$

$$\lim_{x \rightarrow \infty} 0 =$$

For the function  $f(x)$  sketched at the right, evaluate the following limits:

$$\lim_{x \rightarrow \infty} f(x) =$$

$$\lim_{x \rightarrow -\infty} f(x) =$$

$$\lim_{x \rightarrow 0} f(x) =$$

$$\lim_{x \rightarrow 2} f(x) =$$

$$\text{BUT } f(2) =$$

Question: What is  $\lim_{x \rightarrow \infty} \left(5 - \frac{1}{x}\right)$ ?

Consider the function  $f(x) = \frac{x^2 - 9}{x - 3}$

$$f(3) =$$

Let's consider a table of values for  $x$  getting closer and closer to 3:

$$x \quad f(x)$$

$$3.1$$

$$3.01$$

$$3.001$$

$$3.0001$$

Based upon this evidence, what is a good guess for  $\lim_{x \rightarrow 3} f(x)$ ?

An algebraic verification of our guess:

$$\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} =$$

A graph of the function  $\frac{x^2 - 9}{x - 3}$  :

Repeat the same procedure (using a table to make a guess, then algebraically confirming your guess,

then drawing a graph with an appropriate “hole” in it) for  $\lim_{x \rightarrow 0} \frac{2x - x^2}{x}$  .

KEY OBSERVATION: It is possible to have a function  $f(x)$  such that  $f(0)$  is undefined, but for which  $\lim_{x \rightarrow 0} f(x)$  exists. Understanding this concept will be crucial to your understanding of the concept of the “derivative” (our ultimate objective).

## 1.11 Average rate of change

Often we are interested in HOW FAST a function is changing.

For example you drive a distance of 45 miles in one hour. Your average speed was:

Consider the following graphs, and interpret the slopes of the lines in terms of an average rate of change. (Note that there are UNITS associated with the slopes.)

If we have a LINEAR function  $f(x)$ , the slope of the line is the “rate of change.”

But what if our function is not linear? We can still determine an AVERAGE rate of change over a given interval.

A person is taking a trip, and the following graph show’s the person’s distance traveled as a function of time.

1. Describe the person’s trip. (Starting at the second hour, what happened for the next hour and a half?)
2. How fast did the person drive the first two hours of the trip?
3. What was the AVERAGE speed for the entire trip?
4. Estimate the average speed for the last 2 hours of the trip.

A rock is dropped off of a cliff. During the first several seconds of free-fall, the distance fallen by the rock is given by the equation  $d = 16t^2$ , where  $t$  = time (in seconds) and  $d$  = distance fallen (in feet). What is the average speed of the rock over the first 2 seconds?

An amount of money is deposited into a mutual fund account. The graph below shows the account's value over a period of 6 years.

How much money was initially deposited into the account?

What was the account's value at the end of 6 years?

What was the average rate of growth over the 6 years? (Write the units with your answer.)

IN SUMMARY,  
the slope of the secant line between two points on a graph gives us the AVERAGE RATE  
OF CHANGE of the function over that time interval.

## 1.12 Instantaneous rate of change: the derivative

Preliminary review of function notation. If  $f(x) = 4x^2$ , then

$$f(2) = \qquad f(z) = \qquad f(Dx) =$$

$$f(x + Dx) =$$

If you are driving down I-40, we know how to compute your AVERAGE speed over an interval of time: distance traveled  $\div$  elapsed time (e.g., miles per hour, feet per second, etc.)

If you look at your speedometer, you read your INSTANTANEOUS speed. How do we compute your instantaneous speed (or equivalently, instantaneous rate of change)? THAT is the concept of the derivative.

A quick review of the average rate of change:

Sketch the function  $f(x) = x^2 + 1$

Draw the secant line between the points  
(0, 1) and (2, 5) on the curve

The slope of the secant line is

The average rate of change of  $f(x)$  from  $x = 0$  to  $x = 2$  is

\*\*\*\*

Now generalize. Let  $Dx$  represent a “small change in  $x$ .”

On the graph of the function drawn at the right, draw the secant line between the points  $(x, f(x))$  and  $(x + Dx, f(x + Dx))$ .

The slope of the secant line is

Again, this represents the AVERAGE RATE OF CHANGE of  $f(x)$  over the interval of width  $Dx$ .

\*\*\*\*

Now draw the same curve, and focus on the same point  $(x, f(x))$ .

Again consider a secant line between  $(x, f(x))$  and a nearby point  $(x + Dx, f(x + Dx))$ .

Now let  $Dx$  shrink to 0. As the second point slides down the curve towards the first point, the secant lines approach a fixed line called the TANGENT LINE.

THE DERIVATIVE OF THE FUNCTION  $f(x)$  IS THE SLOPE OF THE TANGENT LINE AT  $x$ . IT CAN BE INTERPRETED AS THE “INSTANTANEOUS RATE OF CHANGE OF  $f(x)$  AT  $x$ .”

Notation: Let  $y = f(x)$ . The derivative of the function is sometimes denoted  $f'(x)$  and sometimes denoted  $\frac{dy}{dx}$ .

Formal definition:  $f'(x)$  or  $\frac{dy}{dx} =$

Examples:

1)  $f(x) = x^2 + 1$ . Evaluate the derivative  $f'(x)$ . Evaluate  $f'(0)$ ,  $f'(1)$ , and  $f'(2)$ , and interpret each of these values.

2)  $g(x) = 3x - 1$ . Find the derivative  $\frac{dy}{dx}$ . Does the answer surprise you?

3)  $f(x) = 2x + 6$ . Find  $\frac{dy}{dx}$ .

4)  $g(x) = 3x^2 - 4x$ . Evaluate  $g'(x)$ .

## 1.13 Derivatives, part 2

Shortcut formula: if  $f(x) = x^n$ , then  $f'(x) = nx^{n-1}$

Examples:

$$f(x) = x^3 \qquad f'(x) =$$

$$f(x) = x^4 \qquad f'(x) =$$

$$f(x) = x^{100} \qquad f'(x) =$$

$$f(x) = x \qquad f'(x) =$$

To understand why this pattern holds, let's discuss the binomial theorem:

$$\begin{aligned}(x + y)^2 &= x^2 + 2xy + y^2 \\(x + y)^3 &= x^3 + 3x^2y + 3xy^2 + y^3 \\(x + y)^4 &= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4\end{aligned}$$

and in general,

$$(x + y)^n = x^n + nx^{n-1}y + \text{a bunch of terms containing } y \text{ to a power greater than } 1$$

Review the definition of the derivative  $f'(x)$  :

Let  $f(x) = x^n$ . Then  $f'(x) =$

$$= nx^{n-1}$$

A second rule (given here without proof) that makes life simpler:

Let “ $c$ ” be a constant (i.e., a number). The derivative of  $cf(x) = cf'(x)$ .

Examples:

$$f(x) = 4x^5 \qquad f'(x) =$$

$$f(x) = 20x^3 \qquad f'(x) =$$

$$f(x) = 2x^{100} \qquad f'(x) =$$

$$f(x) = 3x \qquad f'(x) =$$

One more rule (given here without proof) to help us even more:

If  $y = f(x) + g(x)$ , then its derivative  $\frac{dy}{dx} = f'(x) + g'(x)$ .

Examples:

$$f(x) = 4x^5 + 3x^2 \qquad f'(x) =$$

$$y = 9x^{10} + 1000x \qquad \frac{dy}{dx} =$$

Recall: the derivative  $f'(x)$  can be interpreted as:

1)

2)

Consider the function  $f(x) = 3$  (or equivalently, the line  $y = 3$ ).

Sketch the function in the space at the right.

What is the slope of the line?

Thus, the derivative of the function is

**THUS ... the derivative of a constant  $c$  is zero.**

Examples:

$$f(x) = 1000 \qquad f'(x) =$$

$$f(x) = 6x^6 + 5x - 3 \qquad f'(x) =$$

Problems:

1)  $f(x) = x^3$

What is the slope of the line tangent to the curve at  $x = 2$ ?

Write an equation of the line tangent to the curve at the point  $(2, 8)$ .

2) A rock is dropped off of a cliff. For the first few seconds of freefall, the distance the rock has fallen is given by the function  $d(t) = 16t^2$ , where  $t$  is in seconds and  $d$  is in feet. How far has the rock fallen after 2 seconds, and how fast is it falling at that time?

## 1.14 An application of derivatives to economics

Revenue: total money received for goods or services

if selling goods, Revenue = number of items sold  $\times$  selling price

Profit: Revenue - Cost

Example: You make 10 whimmydiddles at a cost of \$10 each. You sell them for \$25 apiece.

Revenue =

Cost =

Profit =

Now let  $x$  be the quantity of whimmydiddles that you make and sell. Find Revenue, Cost, and Profit as a function of  $x$ :

$R(x) =$

$C(x) =$

$P(x) =$

Economists use the term “marginal profit” to mean approximately “the increase in profit if I sell one more unit.” Likewise, marginal revenue and marginal cost are the approximate increase in revenue and cost if we sell one more unit. More specifically, if we have Revenue,

Cost, and Profit as a function of the quantity of items sold  $x$ , then marginal revenue, marginal cost, and marginal profit are the derivatives of these 3 functions, respectively.

In the above example,

marginal revenue =

marginal cost =

marginal profit =

Another example:

Suppose that the profit function (profit as a function of number of items sold) is given by

$$P(x) = -x^2 + 200x - 1000$$

What is the profit if we sell 50 items?

What is the marginal profit?

What is the marginal profit when  $x=50$ ?

Next example:

The number of widgets we can sell is a linear function of the price. If we charge \$10, the price is too high, and there is no demand. If we give them away for free, we can get rid of 10,000.

$$x = 10000 - 1000p$$

Economists usually use the quantity  $x$  as the independent variable. So let's take the above equation and solve for  $p$  in terms of  $x$ :

Recall that Revenue = number of items sold  $\times$  price per item.

$$\text{Revenue } R(x) = xp =$$

Suppose, further that we can produce widgets at a cost of \$1 apiece.

$$\text{Cost function } C(x) =$$

$$\text{Profit function } P(x) =$$

Determine the revenue, cost, and profit if we are currently selling 4000 widgets.

Determine the marginal revenue, marginal cost, and marginal profit.

What is the marginal revenue, marginal cost, and marginal profit if we are currently selling 4000 widgets?

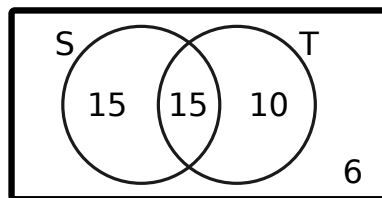
What is the marginal profit if we are currently selling 5000 widgets?

## 2 Homework Exercises - Probability and Statistics

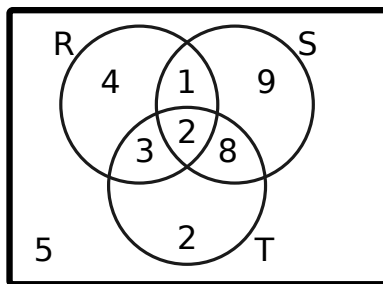
Supplementary problems in probability and statistics.

### 2.1 Venn diagrams

- Referring to the Venn Diagram below, determine the number of elements in each of the following sets:
  - number of elements in  $S$
  - number of elements in  $T$
  - number of elements in  $S \cup T$
  - number of elements in  $S \cap T$
  - number of elements in  $\bar{S}$
  - number of elements in  $S \bar{T}$
  - number of elements in  $S \bar{T}$



- Referring to the Venn diagram below, determine the number of elements in each of the following sets:
  - number of elements in  $S$
  - number of elements in  $R \cap S \cap T$
  - number of elements in  $R \cap \bar{S} \cap T$
  - number of elements in  $R \cap S$
  - number of elements in  $S \cap T$
  - number of elements in  $R \cup S$
  - number of elements in  $\bar{S}$
  - number of elements in  $S \bar{T}$
  - number of elements in  $S \bar{T}$



3. Draw a Venn diagram to represent the following:  
 There are 50 azaleas in a nursery. 20 of the azaleas are both large-flowered AND evergreen.  
 30 of the azaleas are large-flowered (of which 20, recall, are evergreen).  
 35 of the azaleas are evergreen (of which 20, recall, are large-flowered).  
 Letting E represent the set of evergreen azaleas and L the set of large-flowered, draw a Venn diagram representing this situation, labeling each part of the picture with the appropriate number.  
 How many of the azaleas are neither evergreen nor large-flowered?
4. The same nursery has 100 trees.  
 Some particular tree characteristics of interest are Dwarf (D), Broadleaf (B), and Evergreen (E).  
 10 of the trees have all 3 of these characteristics (B and D and E).  
 20 (including the above 10) are both Dwarf and Broadleaf.  
 22 are both Dwarf and Evergreen, and  
 18 are both Broadleaf and Evergreen.  
 47 of the trees are Dwarf (including many of the trees already counted above),  
 53 of the trees are Broadleaf, and  
 50 of the trees are Evergreen.  
 Draw a Venn Diagram, labeling each part of your picture with the appropriate number.

## 2.2 Probability problems (deck of cards)

There are 52 cards in a deck of cards (excluding jokers), as follows:

There are 4 suits: clubs, diamonds, hearts, and spades.

Hearts and diamonds are RED; clubs and spades are BLACK.

There are 13 cards in each suit: two, three, four, ..., nine, ten, jack, queen, king, ace.

(Thus, there are 26 red cards and 26 black cards.)

Jacks, queens, and kings are collectively called FACE CARDS.

Consider an experiment in which a card is chosen randomly from an ordinary deck of cards. (Thus, the sample space contains 52 possible outcomes.) Find the following probabilities:

1.  $P(\text{card is an ace})$
2.  $P(\text{card is a heart})$
3.  $P(\text{card is both an ace AND a heart})$
4.  $P(\text{card is a RED card})$
5.  $P(\text{card is a FACE card})$
6.  $P(\text{card is a red card AND a face card})$
7. Question: Are the following events disjoint? EXPLAIN why or why not.

Event A = the event that the card is a red card

Event B = the event that the card is a club

answer:

8. What is  $P(\text{card is a red card AND the card is a club})$
9. Question: Are the following events disjoint? EXPLAIN why or why not.

Event A = the event that the card is a red card

Event B = the event that the card is an ace

answer:

10. CAREFULLY count the number of cards which are either red cards OR aces, being careful not to count any cards twice. Use your result to answer the following question: QUESTION: What is  $P(\text{card is a red card OR card is an ace})$
11. According to the “addition rule” on the bottom of page 128 of the text,  $P(E \text{ or } F) = P(E) + P(F)$ .

But if you look at your answers to questions 1, 4 and 10, you will note that the formula DOES NOT WORK. Explain why.

12. Find  $P(\text{card is a face card} \mid \text{card is a heart})$

## 2.3 Probability problems (pair of dice)

The roll of a pair of dice

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Consider an experiment in which a PAIR of dice is rolled. An equiprobable sample space (meaning that all of the outcomes are equally likely) is illustrated above.

Observe that there are 36 outcomes in this sample space.

Explanation: 3,1 represents a roll of three on the first die and a one on the second die, etc. Look at the “upward diagonal” of pairs (3,1) (2,2) (1,3) in the diagram. Notice that these are

precisely the possibilities of rolling a SUM of 4.

Then notice that all of the other “upward diagonals” represent rolls of a fixed sum.

A pair of dice is rolled. Find the following probabilities:

(Leave answers in fraction form. It is not necessary to reduce the fractions.)

1. P(the sum of the dice is 2)
2. P(the sum of the dice is 3)
3. P(the sum of the dice is 7)
4. P(The sum of the dice is 12)
5. Which sum is the most likely to occur?

6.  $P(\text{the sum of the dice is 12, GIVEN that the first roll of the die was a 6}).$   
{This could be written as  $P(\text{sum} = 12 \text{ — first roll} = 6)$  }

7.  $P(\text{the sum of the dice is 13})$

8.  $P(\text{the sum of the dice is LESS than 13})$

9.  $P(\text{the sum of the dice is greater than or equal to 11})$

10.  $P(\text{the sum of the dice is 11 — the first roll was a 4})$

11.  $P(\text{the first roll was a 5 — the sum of the dice is 8})$

12.  $P(\text{either the sum of the dice is 3 OR the sum of the dice is 4})$

13.  $P(\text{the first die is a 1 AND the second die is a 5})$

14. QUESTION: are the following events disjoint (mutually exclusive)?

Why or why not?

Event A = the event that the first roll is a 1

Event B = the event that the second roll is a 5

15. QUESTION: are the events A and B in question 14 INDEPENDENT?

Why or why not?

## 2.4 Contingency tables, part 1

1. A tray has 120 candies on it. The candies are categorized according to the cross-table below.

	Chocolate	Peppermint
hard	10	60
soft	40	10

A candy is selected at random from the tray. Find the following probabilities:

- (a)  $P(\text{the candy is chocolate})$
- (b)  $P(\text{the candy is soft})$
- (c)  $P(\text{the candy is both hard AND peppermint})$
- (d)  $P(\text{the candy is either hard OR peppermint})$
- (e)  $P(\text{the candy is hard, GIVEN that it is peppermint}), \text{ i.e. } P(\text{hard} \mid \text{peppermint})$
- (f)  $P(\text{the candy is peppermint, GIVEN that it is hard}), \text{ i.e. } P(\text{peppermint} \mid \text{hard})$
- (g) Are the events “the candy is hard” and “the candy is peppermint” disjoint (mutually exclusive)?
- (h) Are the events “the candy is hard” and “the candy is peppermint” independent?

2. Some children are going to select a game from among 15 games, which have been categorized according to the cross table below.

	fun	boring
active	10	0
inactive	2	3

A game is selected at random from among the 15. Find the following probabilities:

- (a)  $P(\text{the game is boring})$
  
- (b)  $P(\text{the game is active})$
  
- (c)  $P(\text{the game is both active AND boring})$
  
- (d)  $P(\text{the game is either inactive OR boring})$
  
- (e)  $P(\text{the game is fun, GIVEN that it is inactive}), \text{ i.e. } P(\text{fun} \mid \text{inactive})$
  
- (f)  $P(\text{the game is inactive, GIVEN that it is fun}), \text{ i.e. } P(\text{inactive} \mid \text{fun})$
  
- (g) Are the events “the game is active” and “the game is boring” mutually exclusive?
  
- (h) Are the events “the game is active” and “the game is boring” independent?

3. Sixty plastic balls are in a pen. b.They are categorized according to the cross-table below.

	large	small
red	10	10
white	20	20

If a ball is randomly selected, find

- (a)  $P(\text{the ball is red})$
  
- (b)  $P(\text{the ball is red GIVEN that it is large}), \text{ i.e., } P(\text{red} \mid \text{large})$
  
- (c) Are the events “the ball is red” and “the ball is large” independent?

## 2.5 Contingency tables, part 2

In a study to determine the effectiveness of a sleeping drug, some patients were given the drug and some were given a placebo. The patients were then monitored to determine who fell asleep within the next hour. The results were as follows:

	given drug	given placebo
slept	75	60
did not sleep	25	20

If a patient is selected at random, what is the probability that:

1. the patient slept?
2. the patient did NOT sleep?
3. the patient was given the drug?
4. the patient was NOT given the drug?
5. the patient both slept AND was given the drug?
6. the patient did not sleep AND was given a placebo?
7. the patient either slept OR was given the drug?
8. the patient either slept OR was given a placebo?
9. the patient slept, GIVEN that the patient was given the drug?

10. the patient slept, GIVEN that the patient was given a placebo?

11. the patient was given the drug, GIVEN that the patient slept?

12. Compare your answers to problems 1, 9, and 10. Based on your answers, do you think that the drug is effective?

## 2.6 Multiplication rule for independent events

IF events A and B are independent, then  $P(A \text{ and } B) = P(A) \times P(B)$ .

1. A fair coin is tossed, followed by a roll of a 6-sided die. Find the following probabilities:

- a.  $P(\text{the coin is heads AND the die is a six})$
- b.  $P(\text{the coin is heads AND the die is an even number})$
- c.  $P(\text{the coin is tails AND the die is less than 5})$
- d.  $P(\text{the coin is tails AND the die is less than 7})$

2. One hat contains five slips of paper with the letters A, B, C, D, and E.

An urn contains ten slips of paper with the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

A slip of paper is selected at random from the hat, and a slip of paper is selected from the urn.

Find the following probabilities:

- a.  $P(\text{a vowel AND an even number are selected})$
- b.  $P(\text{a consonant AND an odd number are selected})$
- c.  $P(\text{a vowel is selected AND a number greater than 4 is selected})$

3. A machine has a horn and a light, each of which works independently of the other. The horn has a 98% chance of working, and the light has a 95% chance of working. Find the following probabilities:

- a.  $P(\text{the horn works AND the light works})$
  
- b.  $P(\text{at least one of the 2 components fails to work})$   
{Hint: how are the events in questions a and b related?}
  
- c.  $P(\text{the horn fails AND the light works})$
  
- d.  $P(\text{the horn fails AND the light fails})$

4. A string of Christmas lights has 10 lights on it. Each light works independently of the others.

When the string is first plugged in, each light has a 98% probability of working.

- a. What is the probability that all 10 of the lights work?  
This is  $P(\text{first light works AND second light works AND third light works AND ... AND the tenth light works})$
  
- b. What is the probability that AT LEAST ONE light FAILS to work?  
{Hint: how are the events in part a and part b related?}

## 2.7 Extra problems on the normal distribution, part 1

Suppose the scores on a national test are normally distributed with a mean  $m = 500$  and standard deviation  $s = 100$ .

1. What percent of the people taking the test scored above 600?
2. What percent of people taking the test scored between 450 and 700?
3. What percent of the people taking the test scored within 2 standard deviations of the mean?
4. 67% of the test takers scored below what value?
5. Find two values  $x_1$  and  $x_2$ , symmetric around the mean of 500, such that 95% of the test takers scored between  $x_1$  and  $x_2$ .

## 2.8 Extra problems on the normal distribution, part 2

Suppose that the corn yield (per acre) in a particular area is normally distributed with a mean yield of 130 bushels per acre and a standard deviation of 5 bushels per acre.

For each of the following problems, draw a picture and then answer the question.

1. Find the probability that a randomly selected acre has a yield of less than 127 bushels.
2. Find the probability that a randomly selected acre has a yield of more than 127 bushels.
3. Find the probability that a randomly selected acre has a yield of between 125 and 140 acres.
4. 88.88% of all acres in the area have a yield less than what value?
5. 95% of all acres yield between \_\_\_\_\_ and \_\_\_\_\_ bushels.  
(These values should be symmetric around the mean of 130.)

### 3 Homework Exercises

Algebra skills, functions, and rates of change

#### 3.1 Brush up on your equation-solving skills

“Brush up” on your equation-solving skills

Solve for  $x$ :

1.  $4x + 35 = 23$

2.  $7x - 10 = 25$

3.  $5x + 12 = 38$

4.  $\frac{x + 4}{3} = 15$

5.  $\frac{2x - 4}{5} = 6$

6.  $\frac{3x + 2}{4} = 7$

7.  $\frac{x + 6}{2} = \frac{2x + 4}{3}$

8.  $\frac{2x - 1}{2} = \frac{4x + 3}{5}$

9.  $\frac{2x + 5}{3} = \frac{3x - 1}{2}$

## 3.2 Functions

- For each of the following sets of ordered pairs, make a graph of the points on the x-y plane (do NOT try to “connect the dots”). Determine whether or not the set represents a function, and if so determine the domain and range:
  - $\{(1, 0) (2, 2) (3, 4) (4, 6) (5, 8)\}$
  - $\{(1, 3) (2, 3) (3, 3) (4, 3)\}$
  - $\{(1, 1) (1, 2) (1, 3) (1, 4)\}$
  - $\{(-2, 2) (-1, 1) (0, 0) (1, 1) (2, 2)\}$
  - $\{(-1, -1) (-1, 1) (1, -1) (1, 1)\}$
- For each of the following equations, let  $x$  be the “input” variable and let  $y$  be the “output” variable.  
By plotting several points and connecting them with a line or curve, sketch a graph of each of the following functions.  
(The first three functions are LINEAR, i.e., their graphs will be straight lines. The remaining functions are not linear. You will soon be learning how to recognize linear functions.)
  - $y = 3x$
  - $y = 2x$
  - $y = 4 - x$
  - $y = x^2 + 1$
  - $y = \sqrt{x}$
  - $y = x^3$
- Determine the domains and ranges of the functions in parts a, d, and e of question 2.
- The area of a circle is a function of its radius. In particular,  $A = pr^2$ , where  $r$  (the “input variable”) represents the circle’s radius and  $A$  (the “output” variable”) is the circle’s area. Find the areas of the circles with the following radii (plural of radius). Give an exact answer. (Specifically, leave  $p$  in your final answer, as opposed to giving a decimal answer.) Note the units that are given in the final answer.
  - the radius of the circle is 2 inches
  - the radius of the circle is 5 cm.
  - the radius of the circle is 1 foot.

5. John drives from the Biltmore Village exit on I-40 (exit 50 in Asheville) down to the Jamestown Road exit (exit 100 in Morganton).

If you understand the method that North Carolina uses to designate interstate exit numbers, you will know that the total distance of this trip is 50 miles. (Beware! Not all states use the same system for numbering interstate exits.)

Suppose John drove at a constant rate of 50 miles per hour (thus making his trip 1 hour long).

Make a graph of

- (a) John's distance from the Biltmore Village exit as a function of time.
- (b) John's distance from the Jamestown Road exit as a function of time.
- (c) John's distance from exit 75 as a function of time. (This exit is halfway between Asheville and Morganton.)
6. A potato is taken out of a  $400^\circ$  oven and placed outside in a blanket of  $20^\circ$  snow. (In this problem, all temperatures are measured in degrees Fahrenheit.) Initially at a temperature of  $400^\circ$ , the potato naturally cools off, and its temperature eventually approaches  $20^\circ$ . At first, the potato cools off very rapidly. But as the potato's temperature gets closer and closer to  $20^\circ$ , it cools down much more slowly. Make a sketch of the potato's temperature as a function of time.
- (This is an example of Newton's Law of Cooling. While there is a BIG difference between the temperature of some object and the temperature of its surroundings, the object will either heat up or cool down very quickly. But when there is a small difference between the object's temperature and the surrounding temperature, the object does not heat or cool very quickly.)
7. A  $40^\circ$  casserole is taken from the refrigerator and placed in a  $300^\circ$  oven. Applying the principle stated in the above problem, make a sketch of the casserole's temperature as a function of time.
8. Mama Jo Leadfoot also drove from Asheville to Morganton. The details of her woeful tale are given below. After reading the story, you are to make a sketch of Mama Jo's SPEED as a function of time.

Mama Jo's sad story:

For the first 10 minutes of the trip, she drove at a constant rate (not over the speed limit).

During the second 10 minutes of the trip, she got bored and her foot got

“heavy” on the accelerator.

She drove faster and faster and faster ... right through a speed trap!

For the next 10 minutes of the trip, Mama Jo didn't move. She was by the side of the road while a patrolman wrote her a ticket.

For the remaining 30 minutes of the trip, Mama Jo drove at a constant speed, making sure not to drive over the speed limit.

### 3.3 Function notation

1. If  $f(x) = \sqrt{x+2}$  and  $g(x) = \sqrt{x} + 2$ , evaluate  
a.  $f(2)$    b.  $f(7)$    c.  $f(-2)$    d.  $g(1)$    e.  $g(9)$    f.  $g(2)$    h.  $g(7)$    i.  $f(z)$    j.  $g(z)$

k. fill in the blanks:  $f(\underline{\quad}) = \sqrt{3}$  and  $g(\underline{\quad}) = 2$ .

2. If  $g(x) = \begin{cases} x^2 & \text{if } x < 5 \\ x - 5 & \text{if } x \geq 5 \end{cases}$  evaluate

- a.  $g(-2)$    b.  $g(0)$    c.  $g(4)$    d.  $g(5)$    e.  $g(10)$

3.  $f(x) = \begin{cases} 2x & \text{if } x < -2 \\ 4.75 & \text{if } -2 \leq x \leq 2 \\ x^3 & \text{if } x > 2 \end{cases}$

Evaluate:

- a.  $f(-4)$    b.  $f(-2)$    c.  $f(0)$    d.  $f(2)$    e.  $f(3)$

4. Make a sketch of each of the following functions:

a.  $f(x) = 3x - 3$    b.  $h(x) = x^2 - 2$

c.  $f(x) = \begin{cases} 2 & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$    d.  $g(x) = \begin{cases} x & \text{if } x \leq 4 \\ 0 & \text{otherwise} \end{cases}$    g(x)

5. On your calculator you will find the functions  $\ln(x)$  and  $e^x$ . Use these keys to evaluate the following, rounding to a few decimal places. If your calculator does not have the  $\ln(x)$  and  $e^x$  functions, then skip parts a-d, and just work parts e and f.

a.  $\ln(5)$    b.  $e^3$    c.  $\sqrt{e^4 + 1}$    d.  $\ln(2 + e^2)$    e.  $\frac{\sqrt{2} + 5}{3}$

f. If  $f(x) = \frac{10 - \sqrt{x}}{x}$ , evaluate  $f(2)$  and  $f(10)$ .

6. Write a function (any function)  $f(x)$  with the property that  $f(3) = 8$ .

7. Draw a function which has ALL of the following properties:

the domain is  $0 \leq x \leq 4$ ,  
the range is  $0 \leq f(x) \leq 8$ ,  
 $f(0) = f(4)$ , and  
 $f(2) = 8$ .

8. Recall that the area of a rectangle =  $l \times w$  and the perimeter =  $2l + 2w$ , where  $l$  is the length and  $w$  is the width of the rectangle.

If a rectangle is to have a length of 5 inches,

- a) express the rectangle's area  $A$  as a function of its width  $w$
- b) express the rectangle's perimeter  $P$  as a function of its width  $w$
- c) express the rectangle's perimeter  $P$  as a function of its area  $A$ .

### 3.4 Linear functions, part 1

1. Determine which of the following tables represent linear data. For those which are linear, determine the next entry in the table, and find an equation of the line through the data.

x	y	x	y	x	y	x	y
0	2	0	2	0	5	0	0
1	7	1	4	1	6	1	1
2	12	2	5	2	7	4	2
3	17	3	8	3	8	9	3
4	22	4	10	4	9	16	4

2. For each of the following pairs of points, find the slope of the line through the points and find an equation of the line through the points.

- a.  $(0, 0)$  and  $(3, 12)$
- b.  $(-1, 2)$  and  $(2, 5)$
- c.  $(0, 5)$  and  $(1, 8)$
- d.  $(2, 10)$  and  $(6, 2)$

3. Allison can produce 10 whizmos for \$60 and 15 whizmos for \$70.

If the production cost ( $y$ ) is a linear function of the number of whizmos produced ( $x$ ):

- a. Find a linear equation relating the number of whizmos ( $x$ ) to the production cost ( $y$ ).
- b. What is the cost of producing 50 whizmos?
- c. Interpret the slope of the equation in the context of this problem.
- d. What does the  $y$ -intercept represent in this problem?

4. It costs \$4000 to manufacture 10 chairs in one day and \$6000 to manufacture 20 chairs in one day. Assume that the relationship between the number of chairs manufactured in a given day ( $x$ ) and the cost ( $y$ ) is linear.
- Find a linear equation relating the number of chairs manufactured ( $x$ ) to the cost ( $y$ ).
  - What is the cost of manufacturing 3 chairs?
  - Interpret the slope in this problem.
  - Interpret the  $y$ -intercept.

### 3.5 Linear functions, part 2

1. Sketch the following:

(a) a line with  $y$ -intercept  $(0, 5)$  having slope  $= -4$ .

(b) a line through the point  $(1, 1)$  having slope  $= 1/2$ .

(c) a line with  $y$ -intercept  $(0, -1)$  with slope  $= 0$ .

2. Write an equation for each of the lines in question 1.

3. Fill in the missing values in the following table so that the resulting function is linear. Then write an equation for the line through the points.

$x$	$y$
4	1
7	6
13	?
16	?

4. The Acme Gidget-Gadget Company has a fixed operating cost of \$2000 per day. In addition, the production cost is \$5 per gidget-gadget produced.

(a) Letting  $x$  be the number of gidget-gadgets produced in a day, write the total daily operating cost  $C$  as a linear function of  $x$ .

(b) Interpret the slope and  $y$ -intercept in the context of this problem.

(c) What is the total operating cost for a day in which 100 gidget-gadgets are produced?

5. The Acme Whizdoodle Corporation's profit is a linear function of the number of Whizdoodles sold. If 800 whizdoodles are sold, the profit is \$2800. If 1000 whizdoodles are sold, the profit is \$3400.
- (a) Find the profit  $P$  as a linear function of  $x$ .
  - (b) Interpret the slope in the context of this problem.
  - (c) What profit results from the sale of 1500 whizdoodles?
6. Jezebel is driving eastward on I-40 at a constant rate of 60 miles per hour. She started at mile 50. (The mile number increase as you travel eastward.) Let  $t$  represent the time (in hours) that she has driven, and let  $M$  be her mileage point at time  $t$ . Thus,  $M = 50$  when  $t = 0$ .
- (a) Write  $M$  as a linear function of  $t$ .
  - (b) Interpret the slope and  $y$ -intercept in the context of this problem.
  - (c) Where will Jezebel be (at what mile marker) after she has driven 2.5 hours?
7. Jeremiah is also driving eastward on I-40 at a constant rate. One hour after he got on the interstate, he was at mile 92. Three hours after he got on the interstate, he was at mile 222.
- (a) Write his mile number  $M$  as a linear function of time  $t$ .
  - (b) At what exit did Jeremiah get onto I-40, and how fast is he driving?
8. There are 1000 gallons of water in a tank. However, the tank is being drained at a rate of 20 gallons per minute.
- (a) Write the number of gallons of water in the tank  $G$  as a linear function of the elapsed time  $t$  (where  $t$  is in minutes).

- (b) How much water is in the tank after 10 minutes?
  - (c) Interpret the slope and  $y$ -intercept in the context of this problem.
9. The tuition fee for an in-state undergraduate student enrolled at WCU during Summer School 2000 is a linear function of the number of semester hours taken. An in-state student enrolled for 3 semester hours is charged \$294, while a student enrolled in 6 hours is charged \$588.
- (a) Find the tuition charge as a linear function of enrollment hours  $x$ .
  - (b) What is the charge for a student signed up for 4 semester hours?
  - (c) What is the significance of the  $y$ -intercept?

### 3.6 Distributive property and factoring (review)

Multiply each of the following using the distributive property:

1.  $12(x + 3)$
2.  $2x(x - 5)$
3.  $(2x + 4)(x + 5)$
4.  $(2x + 4)(x - 1)$
5.  $(x + 3)2$
6.  $(x - 2)2$
7.  $(2x + 3)2$

FACTOR the following (if possible)

8.  $4x - 16$
9.  $4x^2 - 16x$
10.  $x^2 - 16$
11.  $x^2 + 16$
12.  $x^2 + 9x + 14$
13.  $x^2 - 8x + 15$
14.  $x^2 + 14x - 15$
15.  $5x^2 - 4x - 1$
16.  $3x^2 - 4x + 1$
17.  $4x^2 - 16$
18.  $6x^2 + 13x + 5$
19.  $6x^2 - 13x + 7$
20.  $4x^2 + 25$

### 3.7 Percentages (a quick review)

This is a very short homework set just to make sure that you are comfortable with using “multiplying” factors to effect a percent increase or percent decrease. WORK THESE PROBLEMS USING THE “SHORTCUT” METHOD USED IN CLASS. This homework set should take only a few minutes.

1. A store owner buys an item for \$400 and then marks it up for resale. What is the retail price (markup price) if the markup percentage is:
  - (a) 10%
  - (b) 20%
  - (c) 5%
  - (d) 7.5%
2. An item ordinary sale price is \$400. However, the store is having a markdown sale. What is the discount price (after markdown) if the discount is:
  - (a) 10%
  - (b) 15%
  - (c) 5%
  - (d) 7.5%
3. A store owner purchases an item for \$400. He marks the price up by 15%, resulting in the “retail price.” However, the item doesn’t sell. Some time later, the store has a “15% off” sale. (The retail price is marked down 15%.) What is the “sale price” of the item?  
{Note that the answer is NOT \$400!}

### 3.8 Exponents and exponential growth

1. Without using a calculator, evaluate each of the following:

(a)  $x^4x^6$

(b)  $z^6z^{-2}$

(c)  $\frac{y^7}{y^2}$

(d)  $2^22^3$

(e)  $\frac{3^5}{3^3}$

(f)  $4^{-2}$

(g)  $2^{-4}$

2. Make a sketch of the following functions:  $f(x) = 3^x$  and  $g(x) = 4(.5)^x$

3. Identify which of the following tables represent exponential data, and write a formula for the exponential functions:

x	f(x)	x	g(x)	x	h(x)	x	k(x)
0	4	0	4	0	4	0	0
1	10	1	10	1	1	1	1
2	16	2	25	2	.25	2	4
3	22	3	62.5	3	.0625	3	9

4. \$2000 is deposited into an account earning 8% annual interest. To what amount will the account grow in 5 years if the account is compounded:

(a) annually?

(b) semiannually (twice a year)?

(c) quarterly (4 times a year)?

(d) continuously?

5. How much money must be deposited now into an account earning 10% annual interest, compounded annually, in order for the account to grow to \$2500 after 8 years?
6. How much money must be deposited now into an account earning 8% annual interest, compounded quarterly, in order for the account to grow to \$2500 after 5 years?
7. How much money must be deposited now into an account earning 7.5% annual interest, compounded continuously, in order for the account to grow to \$2500 after 4 years?
8. \$1500 is deposited into an account earning 12% annual interest compounded monthly (12 times a year). To what amount will the account grow to after 6 years?
9. How much money must be deposited now into an account earning 12% annual interest, compounded monthly, in order for the account to grow to \$2500 after 4 years?
10. If there is \$10,000 in an account, and 10% is taken away at the end of every year, how much money remains in the account after 6 years?
11. A bacterial colony begins with 1 single bacterium, and the population doubles every hour. How many bacteria are present after 10 hours? After 24 hours?
12. A radioactive isotope decays at a rate of 1.5% every day. If there is initially 4 grams of the isotope present, how many grams remain after 20 days?

### 3.9 Simplifying rational expressions

Simplify each of the following rational expressions (if possible).

1.  $\frac{a^3b^3c^3}{a^2c^2}$

2.  $\frac{a^3 + b^2}{a + b}$

3.  $\frac{a^2 - b^2}{a + b}$

4.  $\frac{x^2 + 8x + 15}{2x + 10}$

5.  $\frac{x^2 - 3x - 4}{x + 1}$

6.  $\frac{x^2 + 4}{x^2 - 4}$

7.  $\frac{2x^2 - 3x + 1}{3x - 3}$

8.  $\frac{4x + 16}{x + 2}$

9.  $\frac{4x + 16}{x + 4}$

### 3.10 Informal introduction to limits

1. Evaluate  $\lim_{x \rightarrow \infty} f(x)$  for the following functions:

1.  $f(x) = \left(\frac{1}{5}\right)^x$       2.  $f(x) = 10^x$       3.  $f(x) = 4 - \frac{1}{x}$

4.  $f(x) = \frac{1}{x^2}$       5.  $f(x) = 5 + \frac{1}{x^2}$

2. Sketch a function  $g(x)$  with the following properties:

$$\lim_{x \rightarrow -\infty} g(x) = 4, \quad \lim_{x \rightarrow 0} g(x) = 2, \quad \text{and} \quad \lim_{x \rightarrow \infty} g(x) = 0$$

3. Algebraically determine the following limits, and make a sketch of the functions involved.

(Every graph should have a “hole” in the appropriate spot.)

1.  $\lim_{x \rightarrow 4} \frac{x^2 - 16}{x - 4}$       2.  $\lim_{x \rightarrow 1} \frac{x^2 - 2x + 1}{x - 1}$       3.  $\lim_{x \rightarrow -1} \frac{x^2 + 4x + 3}{x + 1}$       4.  $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x - 2}$

### 3.11 Average rate of change

1. Lulu drives down the interstate for 3 hours. Her average speed is 60 miles per hour. Sketch a graph of the distance traveled as a function of time, assuming that she drove at a constant rate.
2. Johnboy drives down the interstate for 3 hours. His average speed for the trip is 60 miles per hour. However, he does NOT drive at a constant rate. Sketch a possible graph of his distance traveled as a function of time.
3. LindaLou drives down the interstate for 2 hours, then takes a 1-hour lunch break, and then continues driving for 1 more hour. Her average speed for the entire 4-hour period is 45 miles per hour. Sketch a possible graph of her distance traveled as a function of time.
4. \$100 is deposited into an account. Three years later, the account is worth \$127. What was the average rate of return, in terms of dollars per year?
5. The number of bacteria in a colony over time is given by the equation  $A = 2^t$ , where  $t$  is the elapsed time (in hours) and  $A$  is the amount of bacteria in the colony.
  - (a) Sketch a graph of this function over a period of 5 hours ( $0 \leq t \leq 5$ ).
  - (b) On your graph, draw the secant line between the points  $(0, 1)$  and  $(5, 32)$ .  
{Note that both of these points should be on your graph. If they aren't, then you messed up!}
  - (c) What is the average growth rate of the colony over the first 5 hours ( in terms of bacteria per hour)?
  - (d) what is the relationship between your answer in part c and the secant line that you drew in part b?
6. A stone is dropped off of a cliff. During the first several seconds of its descent, its distance traveled is given by the equation  $d = 4.9t^2$ , where  $t$  is time (in seconds) and  $d$  is distance fallen (in meters). Find the stone's average speed over the first 3 seconds.
7. The number of widgets produced at a factory is given by the equation  $W(t) = 25t$ , where  $t$  is elapsed time (in hours) and  $W(t)$  is the number of widgets produced in  $t$  hours. What is the rate at which widgets are being produced?

### 3.12 Instantaneous rate of change: the derivative

1. (a) Sketch the function  $f(x) = x^2 - 4$ .  
  
(b) Draw the secant line between the points  $(0, -4)$  and  $(3, 5)$ . What is the slope of this line?  

The slope represents the average rate of change of  $f(x)$  between  $x = 0$  and  $x = 3$ .

  
(c) Draw the line tangent to the curve at the point  $(2, 0)$ .  
  
(d) Find the derivative  $f'(x)$  and evaluate  $f'(2)$ . This represents the slope of the tangent line drawn in part c. It is the instantaneous rate of change of  $f(x)$  at  $x = 2$ .
  
2. (a) Sketch the graph of the function  $g(x) = 1 - x^2$ . (This is a parabola which opens downward.)  
  
(b) Draw a secant line whose slope represents the function's average rate of change from  $x = -2$  to  $x = 0$ .  
  
(c) Draw a line whose slope represents the function's instantaneous rate of change at  $x = 1$ .
  
3. Find the derivative  $f'(x)$  of the following:
  - (a)  $f(x) = 10x + 4$
  - (b)  $f(x) = 2x - 5$
  - (c)  $f(x) = x^2 + 3$
  - (d)  $f(x) = x^2 - 7$
  - (e)  $f(x) = 5x^2 + 3x$
  - (f)  $f(x) = 2x^2 + 5x - 1$

### 3.13 Derivatives, part 2

1. Find the derivative  $f'(x)$  of each of the following functions:

(a)  $f(x) = x^7$

(b)  $f(x) = x^4 + 3x^2 - 10x + 3$

(c)  $f(x) = 2x^4 + 5x^3 - x^2 + 6.3x + p$

(d)  $f(x) = 15x^{10} + \frac{1}{4}x^4 - x^3 + 22x - 28$

(e)  $f(x) = 23.7$

(f)  $f(x) = 100x^5 - x^3 + x + 1$

2. A rock is dropped off of a cliff. For the first few seconds of freefall, the distance the rock has fallen is given by the function  $d(t) = 16t^2$ , where  $t$  is in seconds and  $d$  is in feet.

(a) Graph the function  $d(t)$

(b) How far has the rock fallen after 1.5 seconds?

(c) Draw the secant line through the points  $(0, 0)$  and  $(1.5, 36)$ . What is the slope of this line?

(d) What was the rock's average speed during the first 1.5 seconds of freefall?

(e) Draw the line tangent to  $d(t)$  at  $t = 1.5$ . What is the slope of this line?

(f) How fast was the rock falling at time  $t = 1.5$  ?

3. Consider the function  $f(x) = 4 - x^2$ .
- (a) Sketch the function. (Hint: it is parabola which faces downwards.)
  - (b) Sketch the line tangent to the function at the point  $(2, 0)$ .
  - (c) Find an equation of the tangent line drawn in part b.
4. Consider the function  $f(x) = x^4 - 10x + 9$ .
- (a) Evaluate  $f(2)$
  - (b) Find an equation of the line tangent to the curve of  $f(x)$  at  $x = 2$ .
5. Recall that the derivative tells you about the “steepness” of a curve. (The slope of the tangent line tells us how fast the curve is rising or falling at a particular point.)
- (a) Sketch a function  $f(x)$  whose derivative equals 0.
  - (b) Sketch a function  $g(x)$  whose derivative equals 1.
  - (c) Sketch a function  $h(x)$  whose derivative is negative.
  - (d) Sketch a function whose derivative is positive, and whose derivative is increasing (i.e., keeps getting larger and larger).

## 4 Selected answers

### 4.1 Venn diagrams

Assuming that your instructor drew the Venn diagrams in the master course plan, the answers to the first two problems are given.

- 1a. 30
- 1b. 25
- 1c. 40
- 1d. 15
- 1e. 16
- 1f. 6
- 1g. 31
  
- 2a. 20
- 2b. 2
- 2c. 32
- 2d. 3
- 2e. 10
- 2f. 27
- 2g. 14
- 2h. 9
- 2i. 24

## 4.2 Probability problems (deck of cards)

1.  $4/52 = 1/13$
2.  $13/52 = 1/4$
3.  $1/52$
4.  $26/52 = 1/2$
5.  $12/52 = 3/13$
6.  $6/52 = 3/26$
7. Yes. There are NO cards which are both red and clubs.
8. 0
9. No. There are 2 cards which are both red cards and aces.
10.  $28/52$
11. The addition rule for computing  $P(E \text{ or } F)$  only applies if E and F are disjoint events. Since “card is an ace” and “card is a red card” are not disjoint, the rule cannot be used to compute  $P(\text{card is an ace or a red card})$ .
12.  $3/13$

### 4.3 Probability problems (pair of dice)

1.  $1/36$
2.  $2/36$
3.  $6/36$
4.  $1/36$
5. 7
6.  $1/6$
7. 0
8. 1
9.  $3/36 = 1/12$
10. 0
11.  $1/5$
12.  $5/36$
13.  $1/36$
14. No. The intersection of events A and B is not empty; the outcome (1, 5) is a member of both events A and B.
15. Yes.  $P(\text{first roll is a 1}) = P(\text{first roll is a one — second roll is a 5})$ . Both probabilities are  $1/6$ .

#### 4.4 Contingency tables, part 1

1a.  $50/120 = 5/12$

1b.  $50/120 = 5/12$

1c.  $60/120 = 1/2$

1d.  $80/120 = 2/3$

1e.  $60/70 = 6/7$

1f.  $60/70 = 6/7$

1g. No. The intersection of these sets is not empty; there are 60 candies which are both hard and peppermint.

1h. No.  $P(\text{candy is hard}) * P(\text{candy is hard} \text{ --- candy is peppermint})$ .

$P(\text{candy is hard}) = 70/120 = .583$ , whereas

$P(\text{candy is hard} \text{ --- candy is peppermint}) = 60/70 = .857$ .

2a.  $3/15 = 1/5$

2b.  $10/15 = 2/3$

2c. 0

2d.  $5/15 = 1/3$

2e.  $2/5$

2f.  $2/12 = 1/6$

2g. Yes.

2h. No.  $P(\text{active}) = .8$ , whereas  $P(\text{active} \text{ --- boring}) = 0$ . Thus,  $P(\text{active}) * P(\text{active} \text{ --- boring})$ .

3a.  $20/60 = 1/3$

3b.  $10/13 = 1/3$

3c. Yes, since  $P(\text{red}) = P(\text{red} \text{ --- large})$

## 4.5 Contingency tables, part 2

2.  $45/180$
4.  $80/180$
6.  $20/180$
8.  $155/180$
10.  $60/80 = .75$

## 4.6 Multiplication rule for independent events

1a.  $1/2 \times 1/6 = 1/12$

1b.  $1/2 \times 1/2 = 1/4$

1c.  $1/2 \times 4/6 = 4/12 = 1/3$

1d.  $1/2 \times 1 = 1/2$

2a.  $2/5 \times 1/2 = 2/10 = 1/5$

2b.  $3/5 \times 1/2 = 3/10$

2c.  $2/5 \times 6/10 = 12/50 = 6/25$

3a.  $.98 \times .95 = .931$

3b. This is the complement of the event in question a.  $1 - .931 = .069$

3c.  $.02 \times .95 = .019$

3d.  $.02 \times .05 = .001$

4a.  $.98 \times .98 \times .98 \times \dots \times .98 = (.98)^{10} = .817$

4b. complementary event:  $1 - .817 = .183$

## 4.7 Extra problems on the normal distribution, part 1

1.  $P(x > 60) = P(z > 1) = .1587$  Thus 15.87% scored above 600.
2.  $P(450 < x < 700) = P(-.5 < z < 2) = .6687$ . Thus 66.87% scored between 450 and 700.
3.  $P(-2 < z < 2) = .9544$ . Thus, the answer is 95.44%. (Note that this agrees with the “rule of thumb” given by the empirical rule.) In other words, about 95% of the people score between 300 and 700.
4. From the z-table, the z-score of our number is .44.

What value is .44 standard deviations above the mean?  $500 + (.44)(100) = 544$ .

5. If you draw a picture of the normal distribution and shade the area corresponding to an area of .95, then the combined “tails” (unshaded areas) have an area of .05, and thus each of the tails individually has an area of .0250.

The corresponding z-scores are -1.96 and 1.96.

So  $x_1 = 500 - 1.96(100) = 304$

and  $x_2 = 500 + 1.96(100) = 696$ .

## 4.8 Brush up on your equation-solving skills

1.-3

2.  $35/7$

3.  $26/5$

4. 41

5. 17

6.  $26/3$

7. 10

8.  $11/2$

9.  $13/5$

## 4.9 Functions

1.

“a” is a function. Domain =  $\{1, 2, 3, 4, 5\}$ . Range =  $\{0, 2, 4, 6, 8\}$ .

“b” is a function. Domain =  $\{1, 2, 3, 4\}$ . Range =  $\{3\}$ .

“c” is NOT a function.

“d” is a function. Domain =  $\{-2, -1, 0, 1, 2\}$ . Range =  $\{0, 1, 2\}$ .

“e” is NOT a function.

4.

a.  $4\text{p in}^2$  (square inches)

b.  $25\text{p cm}^2$  (square centimeters)

c.  $1\text{ ft}^2$  (square foot)

## 4.10 Function notation

1. a. 2

b. 3

c. 0

d. 3

e. 5

f.  $\sqrt{2} + 2$

g.  $\sqrt{7} + 2$

h.  $\sqrt{z + 2}$

i.  $\sqrt{z} + 2$

j. 1 and 4

2.

a. 4

b. 0

c. 16

d. 0

e. 5

3.

a. -8

b. 4.75

c. 4.75

d. 4.75

e. 27

5.

a. 1.609

b. 20.0855

c. 7.456

d. 2.2395

e. 2.138

6. There are infinitely many correct answers, including  $f(x) = 2x + 2$  or  $f(x) = x^2 - 1$  or  $f(x) = x + 5$  or even  $f(x) = 8$ .

8.

a.  $A = 5w$

b.  $P = 10 + 2w$

c.  $P = 10 + 2A/5$

## 4.11 Linear functions, part 1

1. The first and third tables are linear. Their equations are  $y = 2 + 5x$  and  $y = 5 + x$ , respectively.

2.

a.  $m = 4, y = 4x$

b.  $m = 1, y = 3 + x$

c.  $m = 3, y = 5 + 3x$

d.  $m = -2, y = 14 - 2x$

3.  $y = 40 + 2x$ .

The cost of producing 50 whizmos is \$140.

Interpretation of slope: The cost is \$2 for every ADDITIONAL whizmo produced. (That is, for every additional whizmo produced, the production cost increases by \$2.) Comment: In both interpretations, the word “additional” is critical.

Interpretation of y-intercept: The fixed cost is \$40.

4.  $y = 2000 + 2x$ .

\$2600

The cost increases by \$200 for every additional chair manufactured.

The fixed cost is \$2000.

## 4.12 Linear functions, part 2

2.

a.  $y = 5 - 4x$

b.  $y = .5 + .5x$

c.  $y = -1$

3. Missing values: 16 and 21.

$$y = -(17/3) + (5/3)x$$

4.

a.  $C = 2000 + 5x$

b. slope: The cost increases by \$5 for every additional gadget-gadget produced. y-int: The fixed-cost is \$2000.

c.  $C(100) = \$2500$ .

5.

a.  $P = 400 + 3x$

b. There is an extra \$3 profit for each additional widget sold.

c.  $P(1500) = \$4900$

6.

a.  $M = 50 + 60t$

b. slope: Her speed is 60 miles per hour. y-int: She started at mile 50.

c.  $M(2.5) = 200$

7.

a.  $M = 27 + 65x$

b. He got on at exit 27, driving 65 miles per hour.

8.

a.  $G = 1000 - 20t$ .  $G(10) = 800$ .

b. slope: water is leaving the tank at a rate of 20 gallons per minute. y-int: initially there are 1000 gallons of water in the tank.

9.

a.  $y = 98x$

b.  $y(4) = 392$

c. If a student signs up for 0 credit hours (i.e., no classes are taken), then there is no charge!

### 4.13 Distributive property and factoring (review)

1.  $12x + 36$
2.  $2x^2 - 10x$
3.  $2x^2 + 14x + 20$
4.  $2x^2 + 2x - 4$
5.  $x^2 + 6x + 9$
6.  $x^2 - 4x + 4$
7.  $4x^2 + 12x + 9$
8.  $4(x - 4)$
9.  $4x(x - 4)$
10.  $(x + 4)(x - 4)$
11. does not factor
12.  $(x + 7)(x + 2)$
13.  $(x - 5)(x - 3)$
14.  $(x + 15)(x - 1)$
15.  $(5x + 1)(x - 1)$
16.  $(3x - 1)(x - 1)$
17.  $(2x + 4)(2x - 4)$
18.  $(3x + 5)(2x + 1)$
19.  $(6x - 7)(x - 1)$
20. does not factor

#### 4.14 Percentages (a quick review)

1.a. \$440 b. \$480 c. \$420 d. \$430

2.a. \$360 b. \$340 c. \$380 d. \$370

3. \$391

## 4.15 Exponents and exponential growth

1.

a.  $x^{10}$

b.  $z^4$

c.  $y^5$

d.  $2^5 = 32$

e.  $3^2 = 9$

f.  $1/4^2 = 1/16$

g.  $1/2^4 = 1/16$

3. Functions  $g(x)$  and  $h(x)$  are exponential.

$$g(x) = 4(2.5)^x \text{ and } h(x) = 4(.25)^x$$

4. a.  $A = 2000(1.08)^5 = \$2938.66$

b.  $A = 2000(1.04)^{10} = \$2960.49$

c.  $\$2971.89$

d.  $A = 1000e^{(.08)(5)} = \$2983.65$

5.  $\$1166.27$

6. Solve the equation  $2500 = A_0(1.02)^{20}$ . The answer is  $A_0 = \$1682.43$

7. Solve the equation  $2500 = A_0e^{(.075)(4)}$ . The answer is  $A_0 = \$1852$

8.  $\$3070.65$

9.  $\$1550.65$

10.  $10,000(.9)^6 = \$5314.41$

11. The amount after  $t$  hours  $A(t) = 1(2)^t$ . Thus  $A(10) = 1,024$  and  $A(24) = 16,777,216$ .

12.  $A(t) = 4(.985)^t$ . Thus  $A(20) = 2.96$  grams.

## 4.16 Simplifying rational expressions

1.  $\frac{ab^3}{c^4}$
2. cannot be simplified
3.  $a - b$
4.  $\frac{x + 3}{x + 3}$
5.  $x - 4$
6. cannot be simplified
7.  $\frac{2x - 1}{3}$
8. cannot be simplified
9. 4

## 4.17 Informal introduction to limits

1a. 0   b.  $\infty$    c. 4   d. 0   e. 5

3. The limits are: a. 8   b. 0   c. 2   d. 5

Don't forget to draw the graphs associated with each problem!

## 4.18 Average rate of change

4. AVERAGE rate of return is 9 dollars/year (dollars per year).

5.

c.  $(32 - 1)/5$  is an average growth rate of 6.2 bacteria per hour

d. The average growth rate of 6.2 bacteria per hour equals the slope of the secant line in part b.

6. Note that  $d(0) = 0$  and  $d(3) = 44.1$ . Thus, the stone falls 44.1 meters in 3 seconds.

ANSWER: The AVERAGE speed is 14.7 meters per second

7. 25 widgets per hour!

## 4.19 Instantaneous rate of change: the derivative

1b. slope = 3

d.  $f'(x) = 2x$ . Thus  $f'(2) = 4$ .

3. a. 10

b. 2

c.  $2x$

d.  $2x$

e.  $10x + 3$

f.  $4x + 5$

## 4.20 Derivatives, part 2

1. a.  $7x^6$
- b.  $4x^3 + 6x - 10$
- c.  $8x^3 + 15x^2 - 2x + 6.3$
- d.  $150x^9 + x^3 - 3x^2 + 22$
- e. 0
- f.  $500x^4 - 3x^2 + 1$

2. b.  $d(1.5) = 36$  feet
- c. slope = 24 ft/sec
- d. 24 ft/sec
- e.  $d'(t) = 32t$ . Thus  $d'(1.5) = 48$
- f. 48 ft/sec

3c.  $y = -4x + 8$

- 4.a. 5
- b.  $y - 5 = 22(x - 2)$  or  $y = 22x - 39$

5. a. Any horizontal line
- b. Your graph should be a line whose slope is one.
- c. Your graph could be ANY graph which is decreasing (falling from left to right).